A37240

Calculators may be used in this examination provided they are not capable of being used to store alphabetical information other than hexadecimal numbers

School of Computer Science

First Year Undergraduate

06-35393

LC Theories of Computation

Main Summer Examinations 2025

Time allowed: 2 hours

[Answer all questions]

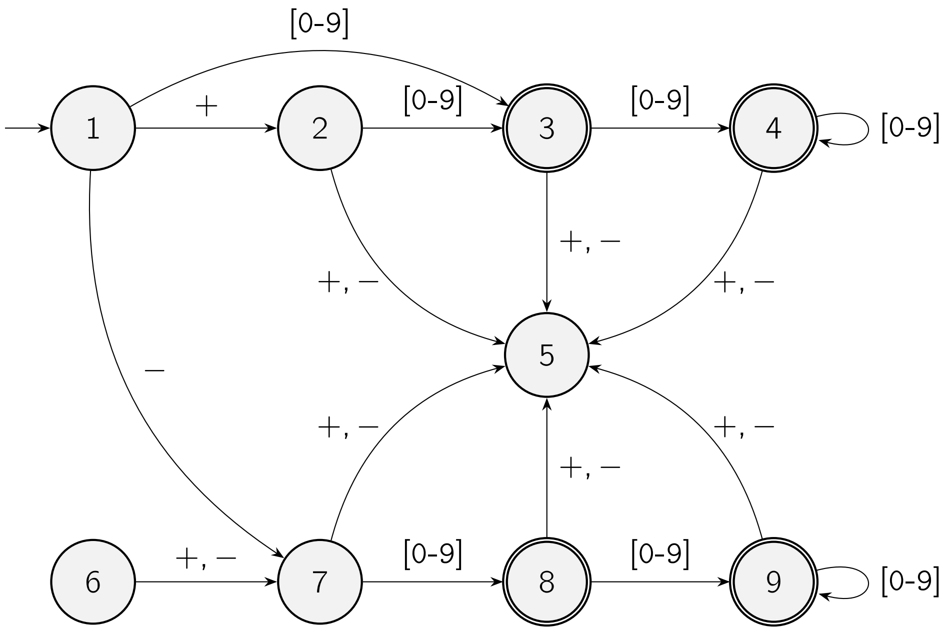
Note

Answer ALL questions.

Question 1 – Languages and Automata [34 marks]

(a) Write a regular expression over the alphabet $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ that recognises all even natural numbers (with leading zeros allowed). For example, 0, 000, 42 and 0585926 should be accepted; while 3, 101 and 074983249 should not. **[8 marks]**

(b) The following partial DFA recognises any integer over the alphabet $\Sigma \cup \{+, -\}$. Note that the [0-9]-transitions accept any digit.



(i) Give every reason why this is *not* a minimal partial DFA. **[8 marks]**

(ii) Draw an equivalent minimal partial DFA. To show that it is equivalent, label each state as the set of all corresponding states of the partial DFA above. **[9 marks]**

(c) Consider the following context-free grammar, which generates a language of simple mathematical expressions:

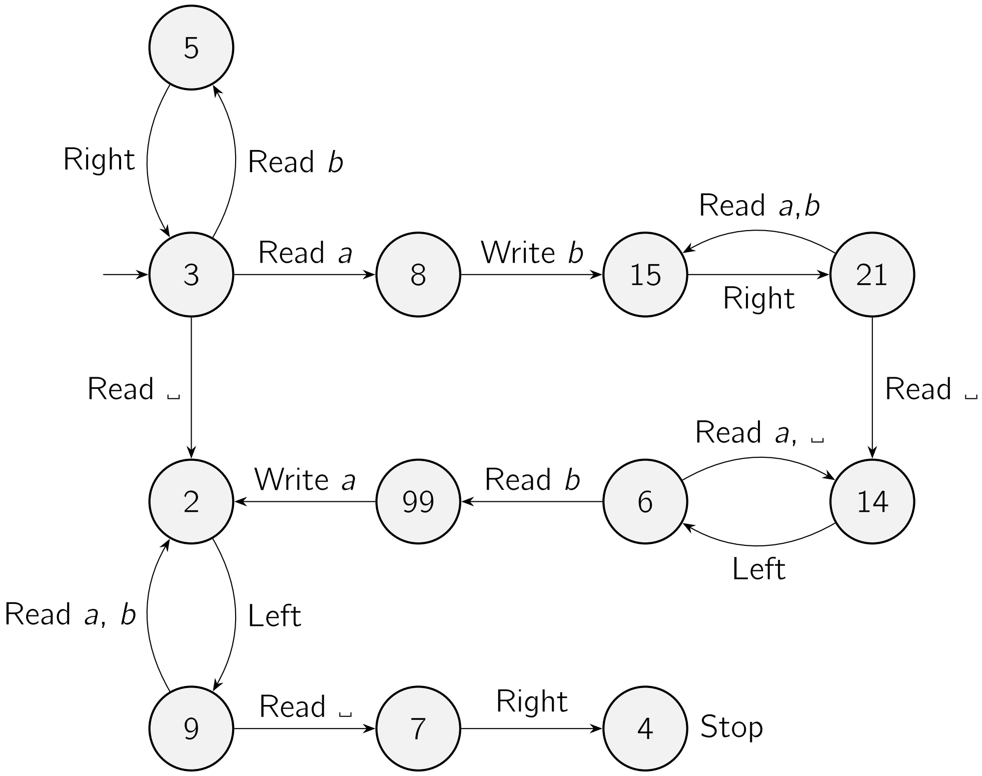
$$\Rightarrow S \;::=\; S + S - 1 \;\mid\; 1 \;\mid\; 0 + S$$

For example, it can generate the expression $1 + 0 + 1 - 1$, which evaluates to 1.

Prove, using structural induction, that the expressions generated by this language never evaluate to a number less than zero. **[9 marks]**

Question 2 – Models and Time [33 marks]

(a) (i) Here is a Turing machine, for the tape alphabet $\{a, b, \sqcup\}$.



The machine starts on the leftmost cell of a nonempty block of a's and b's on an otherwise blank tape. In every case, it ends in the same place, on the leftmost cell of a nonempty block of a's and b's.

Trace the behaviour of the machine with initial tape contents $\dot{a}b$, where the dot indicates the head position. At each stage you should show the tape contents, head position, state, instruction and result in case of a read instruction. (The number of steps is no more than 15.) **[8 marks]**

(ii) In general, describe the machine's output in terms of the given input. That is, depending on the word on the tape initially, you should say what word is on the tape after execution stops. Take care to consider each possible case of the input. **[8 marks]**

(b) Rob wants to convert a fancy Turing machine with tape alphabet $\{a, b, c, \sqcup\}$ to a simple one with tape alphabet $\{a, b, \sqcup\}$. He decides to represent the fancy tape contents by a simple tape contents as follows:

Fancy	Simple	Fancy	Simple
a	a	$\dot{a}$	$\dot{a}$
b	bb	$\dot{b}$	$\dot{b}b$
c	ba	$\dot{c}$	$\dot{b}a$
$\sqcup$	$\sqcup$	$\dot{\sqcup}$	$\dot{\sqcup}$

For example, if the fancy tape contents is $a\dot{b}ac\sqcup b$, then the corresponding simple tape contents is $a\dot{b}baba\sqcup bb$.

Give a simple machine that simulates the Right instruction of the fancy machine. **[8 marks]**

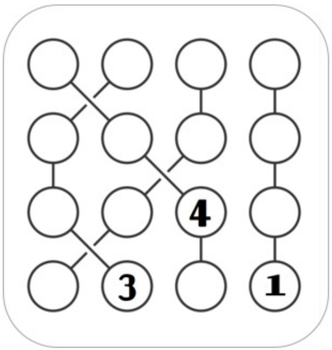
(c) Emma makes a Turing machine with tape alphabet $\{a, b, \sqcup\}$. For an input of length n starting with a, the running time is $5000n^2$ if $n < 100$ and $3n + 700$ otherwise. For an input of length n starting with b, the running time is $n + 17$ if $n < 30$ and $4n + 1$ otherwise. Prove that the worst case running time is $O(n)$. **[9 marks]**

Question 3 – Hard and Impossible Problems [33 marks]

(a) (i) *Strimko* is a puzzle based on a $n \times n$ grid to be filled with numbers from 1 to n (inclusive). Cells in the grid are organized into n streams represented by continuous lines between them. Some numbers are given at the start and some must be added following three basic rules:

- Each row must contain each of the numbers 1 to n.
- Each column must contain each of the numbers 1 to n.
- Each stream must contain different numbers 1 to n.

Below left is an example of a *Strimko* grid of size 4 as given at the start and right is the same grid once filled in.



Explain why the problem of checking a candidate solution for a Strimko puzzle can be accomplished in polynomial time. **[8 marks]**

(ii) It follows that a SAT solver can be used to decide whether a Strimko puzzle is solvable. Explain why. **[8 marks]**

(b) State Rice's theorem. **[8 marks]**

(c) GameGalore is a company that produces video games. Jemima has been asked by the manager to ensure that each game displays the company's logo in every frame. Can she write a program to determine whether this is so? Explain your answer. **[9 marks]**

Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so

Important Reminders

- Coats/outwear should be placed in the designated area.
- Unauthorised materials (e.g. notes or Tippex) must be placed in the designated area.
- Check that you do not have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches **must** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are not permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.
- You are not permitted to have writing on your hand, arm or other body part.
- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately
- Alert an Invigilator immediately if you find any unauthorised item upon you during the examination.

Any students found with non-permitted items upon their person during the examination, or who fail to comply with Examination rules may be subject to Student Conduct procedures.